

• 机械工程 •



本刊网刊

DOI:10.15961/j.jsuese.201901183

Stephenson-III型平面六杆机构轨迹综合的代数求解

李学刚^{1,2}, 张丽娟², 魏世民¹, 李河清²

(1.北京邮电大学 自动化学院,北京 100876; 2.华北理工大学 机械工程学院,河北 唐山 063210)

摘要:为弥补精确点法、优化法和数值图谱法等已有方法的不足,进一步提高Stephenson-III型平面六杆机构连续轨迹综合的精度,提出了一种基于傅氏级数的平面六杆机构轨迹综合的代数求解新方法。通过将Stephenson-III型平面六杆机构拆分为四杆机构和二级杆组,对设计变量进行解耦。在利用已建立综合方法得到左侧四杆机构设计参数的基础上,依据复数矢量理论,建立了含有右侧二杆组设计变量的封闭矢量方程式,将由傅氏级数表示的连杆转角函数代入封闭矢量方程中,经过消元、代换将方程转化为由设计变量、连杆转角函数谐波参数和输入转角表示的复数方程。根据复指数的性质,得到了含有机构设计变量和连杆转角函数谐波参数的表达式,确定了转角函数谐波参数与设计变量间的函数关系。根据这一函数关系建立综合设计方程,利用Groebner基代数法进行消元,将综合设计方程化简为含有机设计参数和连杆转角函数谐波参数一元四次方程,求解得到右侧二杆组设计变量的解析解,建立了右侧二杆组设计变量计算的通用公式。利用该方法进行连杆机构轨迹综合,可以得到左侧四杆机构的12组设计参数以及右侧二杆组的4组设计参数,将所得结果进行组合,最终可以得到48组Stephenson-III型平面六杆机构轨迹综合设计参数。通过运动仿真程序对综合结果进行验证,检验其是否存在曲柄,有无分支、顺序问题,并依据综合误差,最终可得到满足要求的设计参数值。在理论分析的基础上,进一步归纳总结明确了使用该方法进行轨迹综合的具体步骤,利用MATLAB软件编写求解程序,并通过数值实例验证该方法的有效性和可行性。结果表明:该方法实现了Stephenson-III型平面六杆机构的连续轨迹综合,克服了精确点法受机构未知量个数限制,无法实现多点位轨迹综合的不足。与已有数值图谱法和优化法相比,该方法不需要预先建立数值图谱库,也不需要提供优化初值,其通过方程求解得到综合设计结果,具有求解精度高、计算速度快、可重复性强的优点,更加便于计算机编程,为机构综合软件的开发提供了理论基础。

关键词:Stephenson-III型平面六杆机构; 轨迹综合; 傅氏级数; 代数法

中图分类号:TH112

文献标志码:A

文章编号:2096-3246(2021)01-0155-07

Algebraic Solution for Path Synthesis of Planar Stephenson-III Six-bar Linkage

LI Xuegang^{1,2}, ZHANG Lijuan², WEI Shimin¹, LI Heqing²

(1.School of Automation, Beijing Univ. of Posts and Telecommunications, Beijing 100876, China;

2.College of Mechanical Eng., North China Univ. of Sci. and Technol., Tangshan 063210, China)

Abstract: In order to overcome the disadvantage of conventional precise point methods, optimization approaches and numerical atlas, and further improve the accuracy of continuous path synthesis of planar six-bar linkage, a novel analytical approach with Fourier series was presented to solve path synthesis problem for planar six-bar linkage. Firstly, the planar Stephenson-III six-bar linkage was decomposed into a four-bar linkage and a two-bar group for the purpose of decoupling the design parameters of linkage. Then, on the basis of obtaining the design parameters of the four-bar linkage on the left by the established comprehensive method, a closed vector equation containing the design variables of the two-bar group on the right was established based on the complex vector theory. The linkage rotation angle function was formulated according to the Fourier series, and substituted into a vector loop equation. Through eliminating and substituting, the vector loop equation was changed into a complex

收稿日期:2019-12-10

基金项目:国家自然科学基金项目(51375059;51605036);北京市自然科学基金-海淀原始创新联合基金项目(L172031)

作者简介:李学刚(1979—),男,副教授,博士。研究方向:机构学与机器人技术。E-mail: hblgyjs@126.com

网络出版时间:2020-12-17 16:43:47

网络出版地址: <https://kns.cnki.net/kcms/detail/51.1773.TB.20201216.1724.007.html>

number equation that contain the design variables of mechanism, the harmonic parameters of the linkage rotation angle function and the input angle. With regard to the properties of the complex exponent, the mathematic expression containing the design variables of mechanism and the harmonic parameters of the linkage rotation angle function was obtained. Accordingly, the relationship between the design variables and the synthetic design harmonic parameters of rotation angle function was obtained. According to this function relation, the synthetic design equation was established. Through the elimination Groebner base algebra approach, the comprehensive design equation was simplified into a unary quartic equation containing the design parameters of the mechanism and the harmonic parameters of the linkage rotation angle function, the analytical solutions for the two-bar group on the right was derived and the general formula for calculating the design variables of the right two-bar group was established. Using the method of connecting rod mechanism path synthesis, 12 groups of design parameters of the four-bar linkage on the left and 4 groups of design parameters of the two-bar linkage on the right can be obtained. By combining the results, 48 sets of integrated design parameters of planar Stephenson-III six-bar linkage path synthesis can be obtained. Through the dynamic simulation program, the comprehensive results were verified to check whether there were cranks, branching problems and order problems. And the design parameter value which met the requirements can be finally obtained according to comprehensive error. Based on the aforementioned theory, the procedure of solving path synthesis problem by the proposed method can be obtained. The computer programs have been developed for the proposed method by MATLAB for solving the problem. An example was provided to verify the validity and feasibility of the proposed method. Verification results showed that the proposed approach can overcome the shortage of multi-point path synthesis method and to solve path synthesis problems for planar Stephenson-III six-bar linkage with no limitations on the number of precision points. Compared with numerical atlas and optimization approaches, the proposed method avoided the use of extensive numerical atlas databases and optimal initial solutions, and obtained the results by solving the equation. Therefore, this approach has the characteristics of high accuracy, fast solution velocity and high repeatability, and is suitable for computer programming. The research of this approach provided the theory basis for development of synthesis software.

Key words: planar Stephenson-III six-bar linkage; path synthesis; Fourier series; analytical approach

平面六杆机构可以实现丰富的连杆轨迹,与四杆机构相比,其具有运动所占空间小、便于实现停歇运动、易于取得有利传动角、可实现多重运动等优势^[1],但其尺寸参数较多,运动要求多样而复杂,尺度综合设计较为困难。目前,求解该方法的方法主要有图谱法^[2-3]、优化法^[4-7]、精确点法^[8-11]等。其中:图谱法、优化法虽然能够完成平面六杆机构轨迹综合问题,但由于平面六杆机构尺寸参数较多,解域空间大,建立完备的数值图谱库和给定合理优化初值难度较大,使用这些方法进行机构综合有时难以得到精确的最优结果;精确点法通过建立约束方程,求解得到机构尺度参数,虽然可以得到较为精确的综合结果,但受机构未知参数个数的限制,无法实现多点位连续轨迹综合。

作者针对Stephenson-III型平面六杆机构,提出一种基于傅氏级数的连续轨迹综合的代数求解方法。与原有方法相比,该方法通过方程求解得到机构设计参数,其解的精度高,完备性强,且不需要提供优化的初值,也无需事先建立图谱库。有效扩大了代数法的适用范围,提高了连杆机构近似运动综合的求解精度和速度。

1 连杆曲线的傅氏级数表示

图1为Stephenson-III型平面六杆机构示意图。机构各杆长尺寸分别为 l_1 、 l_2 、 l_3 、 l_4 、 l_5 、 l_6 ,机架AD与x轴的夹角为 β ,A点到原点o的距离为 r ,旋转角度为 μ ,

α_1 为机架上的角 $\angle GAD$, α_2 、 α_3 分别为浮动杆I、II上的角 $\angle EBC$ 、 $\angle PEF$, θ_1 为连杆BC与机架AD之间的夹角, θ_2 为连杆EF的辐角, φ 、 φ_0 分别为机构的输入杆转角、初始位置转角,浮动杆I、II上E点、P点分别为连杆轨迹生成点。

Stephenson-III平面六杆机构为多环组合机构,其可以看作在四杆机构ABCD上串联了一个二杆组EFG。其浮动杆I上的E点产生轨迹曲线即为四杆机构ABCD的连杆曲线,浮动杆II上的P点可以产生更为复杂的连杆曲线。

由文献[3]可知:当曲柄AB以 ω 匀速转动时,角度 φ 、 θ_1 、 θ_2 、 ρ 均为以时间 t 为变量的周期性函数,且有 $\varphi = \omega t$;E点和P点产生的连杆曲线为周期性封闭曲线,可以表示为以 φ 为变量的傅氏级数之和:

$$r_E(t) = x_1(t) + iy_1(t) = \sum_{-\infty}^{+\infty} c_n e^{i\varphi} \quad (1)$$

$$r_P(t) = x_2(t) + iy_2(t) = \sum_{-\infty}^{+\infty} b_n e^{i\varphi} \quad (2)$$

同理,连杆转角函数 $e^{i\theta_1}$ 、 $e^{i\theta_2}$ 也可以表示为以 φ 为变量的傅氏级数之和:

$$e^{i\theta_1}(t) = x'_1(t) + iy'_1(t) = \sum_{-\infty}^{+\infty} c'_n e^{i\varphi} \quad (3)$$

$$e^{i\theta_2}(t) = x'_2(t) + iy'_2(t) = \sum_{-\infty}^{+\infty} b'_n e^{i\varphi} \quad (4)$$

式(1)~(4)中: $i = \sqrt{-1}$; c_n 、 b_n 、 c'_n 、 b'_n 分别为 r_E 、 r_P 、 $e^{i\theta_1}$ 、

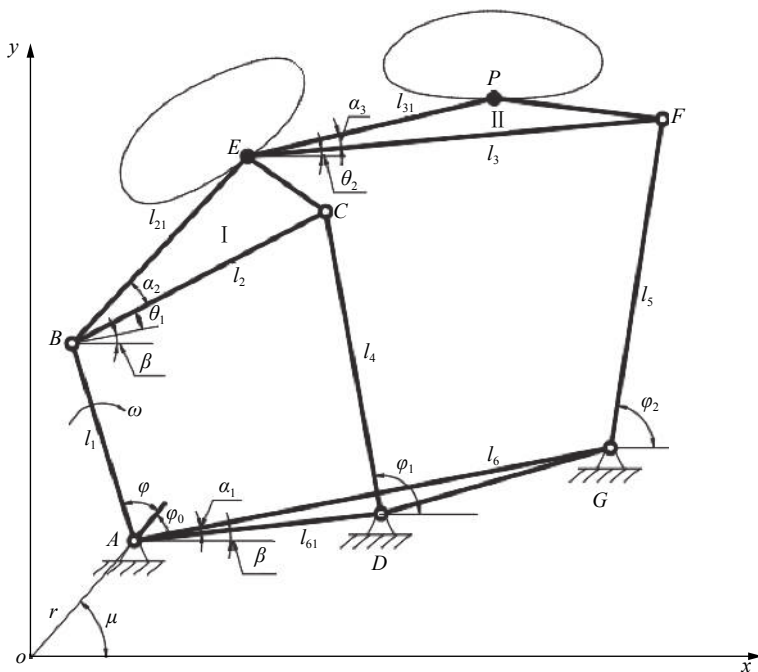


图 1 Stephenson-III 六杆机构轨迹生成图

Fig. 1 Illustration of a planner Stephenson-III six-bar linkage

$e^{i\omega}$ 傅氏级数展开的谐波参数, 且有:

$$c_n = \frac{1}{T} \int_0^T (x_1(t) + iy_1(t)) e^{in\omega t} dt \quad (5)$$

$$b_n = \frac{1}{T} \int_0^T (x_2(t) + iy_2(t)) e^{in\omega t} dt \quad (6)$$

$$c'_n = \frac{1}{T} \int_0^T (x'_1(t) + iy'_1(t)) e^{in\omega t} dt \quad (7)$$

$$b'_n = \frac{1}{T} \int_0^T (x'_2(t) + iy'_2(t)) e^{in\omega t} dt \quad (8)$$

平面六杆机构的连杆曲线为复杂高次曲线, 有时难以用显函数表达, 更多的时候要用一系列的离散点表示, 因此, c_n, c'_n, b_n, b'_n 通常可以用数值方法计算得到. 根据离散傅里叶变换的性质, 由文献[3]可知, c_n, c'_n, b_n, b'_n 的数值解的计算表达式为:

$$c_n = \frac{1}{M} \sum_{m=0}^{M-1} (x_{1m}(t) + iy_{1m}(t)) [\cos(nm\omega) + i \sin(nm\omega)] \quad (9)$$

$$c'_n = \frac{1}{M} \sum_{m=0}^{M-1} (x'_{1m}(t) + iy'_{1m}(t)) [\cos(nm\omega) + i \sin(nm\omega)] \quad (10)$$

$$b_n = \frac{1}{M} \sum_{m=0}^{M-1} (x_{2m}(t) + iy_{2m}(t)) [\cos(nm\omega) + i \sin(nm\omega)] \quad (11)$$

$$b'_n = \frac{1}{M} \sum_{m=0}^{M-1} (x'_{2m}(t) + iy'_{2m}(t)) [\cos(nm\omega) + i \sin(nm\omega)] \quad (12)$$

式中: $n = 0, \pm 1, \dots, \pm(M-1)$; $m = 0, \pm 1, \dots, \pm(M-1)$; $\omega = \pm \frac{2\pi}{M}$; M 为离散点个数。

由文献[12-14]的研究结论可知, 傅氏级数展开的谐波参数 c_n, c'_n 间满足如下关系:

$$c'_n = \frac{c_n e^{-i(\beta+\alpha_2)}}{l_{21}}; n \neq 0, 1 \quad (13)$$

$$c'_0 = \frac{(c_0 - r e^{i\mu}) e^{-i(\beta+\alpha_2)}}{l_{21}} \quad (14)$$

$$c'_1 = \frac{(c_1 - l_1 e^{i\varphi_0}) e^{-i(\beta+\alpha_2)}}{l_{21}} \quad (15)$$

2 平面六杆机构综合设计方程建立

由图1可以发现, 完成 Stephenson-III 型平面六杆机构的轨迹综合, 需要确定包括初始相位角在内的 15 个独立设计变量: $r, l_1, l_2, l_{21}, l_3, l_4, l_5, l_6, l_{61}, \varphi_0, \beta, \alpha_1, \alpha_2, \alpha_3, \mu$, 为使所建立的综合设计方程求解方便, 能够获取方程解析解, 将该机构拆分成 1 个四杆机构和 1 个二杆组, 分两部分求解机构的设计变量。

2.1 左侧四杆机构设计变量求解

如果以 E 点轨迹为综合目标, 机构左侧四杆机构 $ABCD$ 的综合过程可以完全等效为 1 个平面四杆机构轨迹综合问题. 作者在文献[12-14]中已建立了基于傅氏级数的平面四杆机构轨迹综合的代数方法, 提出了该机构轨迹综合设计变量计算的通用公式. 应用该通式可以计算得到左侧四杆机构的 10 个设计变量, $r, \mu, l_1, l_2, l_{21}, l_4, l_{61}, \varphi_0, \alpha_2, \beta$ 变量的计算公式如下:

$$r = \pm \sqrt{ls}, \mu = -i \ln \frac{s}{r}, l_1 = \pm \sqrt{uv}, l_2 = \pm \sqrt{xy}, l_{21} = w,$$

$$l_4 = z, l_{61} = \pm \sqrt{fg}, \varphi_0 = -i \ln \frac{\mu}{l_1}, \alpha_2 = -i \ln \frac{x}{l_2}, \beta = -i \ln \frac{f}{l_{61}}.$$

其中: $s = re^{i\varphi}, l = re^{-i\varphi}, u = l_1 e^{i\varphi_0}, v = l_1 e^{-i\varphi_0}, w = l_{21}, x = l_2 e^{i\alpha_2}, y = l_2 e^{-i\alpha_2}, f = l_{61} e^{i\beta}, g = l_{61} e^{-i\beta}, z = l_4$ 。

2.2 右侧二杆组设计变量求解

在得到左侧四杆机构各设计变量的基础上,如图1所示,应用矢量分析法,建立含有未知设计变量 $l_6, l_3, l_5, \alpha_1, \alpha_3$ 的封闭矢量方程:

$$l_1 + l_{21} + l_3 = l_6 + l_5 \quad (16)$$

方程(16)的复数矢量形式为:

$$l_1 e^{i(\varphi_0 + \varphi_0 + \beta)} + l_{21} e^{i(\alpha_2 + \theta_1 + \beta)} + l_3 e^{i\theta_2} - l_6 e^{i(\alpha_1 + \beta)} = l_5 e^{i\theta_2} \quad (17)$$

取式(17)的共轭可得:

$$l_1 e^{-i(\varphi_0 + \varphi_0 + \beta)} + l_{21} e^{-i(\alpha_2 + \theta_1 + \beta)} + l_3 e^{-i\theta_2} - l_6 e^{-i(\alpha_1 + \beta)} = l_5 e^{-i\theta_2} \quad (18)$$

如图1分析可知,机构连杆转角函数 $e^{i\theta_2}$ 与 $e^{i\varphi}$ 间满足关系为:

$$e^{i\theta_2} = e^{-i\alpha_3} e^{i\varphi} \quad (19)$$

将式(17)和(18)相乘并将式(19)代入,化简可得:

$$\begin{aligned} & uv + hk + cd + ab - o^2 + e^{i\varphi} e^{-i\varphi} uc + e^{-i\varphi} e^{i\varphi} vd - \\ & e^{i\varphi} ud - e^{-i\varphi} va - e^{i\varphi} bd - e^{-i\varphi} ac + e^{i\varphi} e^{-i\theta_1} uk + \\ & e^{-i\varphi} e^{i\theta_1} vh + e^{i\varphi} e^{-i\theta_1} dk + e^{-i\varphi} e^{i\theta_1} ch - e^{-i\theta_1} ak - \\ & e^{i\theta_1} bh = 0 \end{aligned} \quad (20)$$

式中: $a = l_6 e^{i\alpha_1}, b = l_6 e^{-i\alpha_1}, c = l_3 e^{i(\alpha_3 + \beta)}, d = l_3 e^{-i(\alpha_3 + \beta)}, o = l_5, h = l_{21} e^{i\alpha_2}, k = l_{21} e^{-i\alpha_2}$ 。

由式(3)、(4)可知, $e^{i\theta_1}, e^{-i\theta_1}, e^{i\varphi}, e^{-i\varphi}$ 可以表示为以 φ 为变量的傅氏级数之和,由文献[15]可知,利用傅氏级数表示连杆转角函数时,取有限项低次谐波就可较好地表示原函数,取 $n = -3, -2, \dots, 2, 3$ 。将展开后的 $e^{i\theta_2}, e^{-i\theta_2}, e^{i\varphi}, e^{-i\varphi}$ 代入式(20)化简可得:

$$\begin{aligned} & H_{-4} e^{-4i\varphi} + H_{-3} e^{-3i\varphi} + H_{-2} e^{-2i\varphi} + H_{-1} e^{-i\varphi} + H_0 + \\ & H_1 e^{i\varphi} + H_2 e^{2i\varphi} + H_3 e^{3i\varphi} + H_4 e^{4i\varphi} = 0 \end{aligned} \quad (21)$$

式中: $H_{-4} = vdJ_1 + hvJ_2 + hcJ_3 + kdJ_4, H_4 = cu\bar{J}_1 + ku\bar{J}_2 + kd\bar{J}_3 + hc\bar{J}_4, H_{-3} = -adJ_1 - ahJ_2 + dvJ_5 - bcJ_6 + hvJ_7 + hcJ_8 - bkJ_9 + dkJ_{10}, H_3 = -bc\bar{J}_1 - bk\bar{J}_2 + cu\bar{J}_5 - ad\bar{J}_6 + ku\bar{J}_7 + kd\bar{J}_8 - ah\bar{J}_9 + ch\bar{J}_{10}, H_{-2} = -adJ_5 + cuJ_6 - ahJ_7 + kuJ_9 + dvJ_{11} - bcJ_{12} + hvJ_{13} + hcJ_{14} + dkJ_{15} - bkJ_{21}, H_2 = -bc\bar{J}_5 + vd\bar{J}_6 - bk\bar{J}_7 + hv\bar{J}_9 + cu\bar{J}_{11} - ad\bar{J}_{12} + ku\bar{J}_{13} + kd\bar{J}_{14} + hc\bar{J}_{15} - ah\bar{J}_{21}, H_{-1} = -bv - adJ_{11} + cuJ_{12} - ahJ_{13} + vdJ_{16} - bcJ_{17} + hvJ_{18} + hcJ_{19} - bkJ_{20} + kuJ_{21} + kdJ_{22}, H_1 = -au - bc\bar{J}_{11} + vd\bar{J}_{12} - bk\bar{J}_{13} + cu\bar{J}_{16} - ad\bar{J}_{17} + ku\bar{J}_{18} + kd\bar{J}_{19} - ah\bar{J}_{20} + hv\bar{J}_{21} + hc\bar{J}_{22}, H_0 = ab + hk + uv + cd - o^2 - adJ_{16} - bc\bar{J}_{16} + vd\bar{J}_{17} + cuJ_{17} - ahJ_{18} - bk\bar{J}_{18} + hv\bar{J}_{20} + kuJ_{20} + hcJ_{23} + kd\bar{J}_{23}$ 。

其中: $J_1 = b'_{-3}, J_2 = c'_{-3}, J_3 = \bar{b}'_1 c'_{-3} + \bar{b}'_2 c'_{-2} + \bar{b}'_3 c'_{-1}, J_4 = b'_{-3} \bar{c}'_1 + b'_{-2} \bar{c}'_2 + b'_{-1} \bar{c}'_3, J_5 = b'_{-2}, J_6 = \bar{b}'_3, J_7 = c'_{-2}, J_8 = \bar{b}'_0 c'_{-3} + \bar{b}'_1 c'_{-2} + \bar{b}'_2 c'_{-1} + \bar{b}'_3 c'_0, J_9 = \bar{c}'_3, J_{10} = b'_{-3} \bar{c}'_0 + b'_{-2} \bar{c}'_1 + b'_{-1} \bar{c}'_2 + b'_0 \bar{c}'_3, J_{11} =$

$$\begin{aligned} & b'_{-1}, J_{12} = \bar{b}'_2, J_{13} = c'_{-1}, J_{14} = \bar{b}'_1 c'_{-3} + \bar{b}'_0 c'_{-2} + \bar{b}'_1 c'_{-1} + \bar{b}'_2 c'_0 + \bar{b}'_3 c'_1, \\ & J_{15} = b'_{-3} \bar{c}'_{-1} + b'_{-2} \bar{c}'_0 + b'_{-1} \bar{c}'_1 + b'_0 \bar{c}'_2 + b'_1 \bar{c}'_3, J_{16} = b'_0, J_{17} = \bar{b}'_1, \\ & J_{18} = c'_0, J_{19} = \bar{b}'_2 c'_{-3} + \bar{b}'_1 c'_{-2} + \bar{b}'_0 c'_{-1} + \bar{b}'_1 c'_0 + \bar{b}'_2 c'_1 + \bar{b}'_3 c'_2, J_{20} = \\ & \bar{c}'_1, J_{21} = \bar{c}'_2, J_{22} = b'_{-3} \bar{c}'_2 + b'_{-2} \bar{c}'_{-1} + b'_{-1} \bar{c}'_0 + b'_0 \bar{c}'_1 + b'_1 \bar{c}'_2 + b'_2 \bar{c}'_3, \\ & J_{23} = \bar{b}'_3 c'_{-3} + \bar{b}'_2 c'_{-2} + \bar{b}'_1 c'_{-1} + \bar{b}'_0 c'_0 + \bar{b}'_1 c'_1 + \bar{b}'_2 c'_2 + \bar{b}'_3 c'_3. \end{aligned}$$

分析式(21)发现,在表达式 $H_i (i = -4, -3, \dots, 3, 4)$ 中 $H_0, H_1, H_{-1}, H_2, H_{-2}$ 为完整系数表达式,即当 $e^{i\theta_2}, e^{-i\theta_2}$ 和 $e^{i\varphi}, e^{-i\varphi}$ 取更高次谐波时其将不再变化^[16],由复指数的性质可知,这些表达式的值应为0,由此可以得到如下方程:

$$-adJ_5 + cuJ_6 - ahJ_7 + kuJ_9 + dvJ_{11} - bcJ_{12} + hvJ_{13} + hcJ_{14} + dkJ_{15} - bkJ_{21} = 0 \quad (22)$$

$$-bc\bar{J}_5 + vd\bar{J}_6 - bk\bar{J}_7 + hv\bar{J}_9 + cu\bar{J}_{11} - ad\bar{J}_{12} + ku\bar{J}_{13} + kd\bar{J}_{14} + hc\bar{J}_{15} - ah\bar{J}_{21} = 0 \quad (23)$$

$$-bv - adJ_{11} + cuJ_{12} - ahJ_{13} + vdJ_{16} - bcJ_{17} + hvJ_{18} + hcJ_{19} - bkJ_{20} + kuJ_{21} + kdJ_{22} = 0 \quad (24)$$

$$-au - bc\bar{J}_{11} + vd\bar{J}_{12} - bk\bar{J}_{13} + cu\bar{J}_{16} - ad\bar{J}_{17} + ku\bar{J}_{18} + kd\bar{J}_{19} - ah\bar{J}_{20} + hv\bar{J}_{21} + hc\bar{J}_{22} = 0 \quad (25)$$

$$\begin{aligned} & ab + hk + uv + cd - o^2 - adJ_{16} - bc\bar{J}_{16} + \\ & vd\bar{J}_{17} + cuJ_{17} - ahJ_{18} - bk\bar{J}_{18} + hv\bar{J}_{20} + \\ & kuJ_{20} + hcJ_{23} + kd\bar{J}_{23} = 0 \end{aligned} \quad (26)$$

方程(22)~(26)即求解右侧二杆组设计变量的综合方程,其中 u, v, h, k 为含有设计参数的已知变量, a, b, c, d, o 为含有设计参数的未知变量, $c'_n, b'_n (n = -3, -2, \dots, 2, 3)$ 可分别由离散傅里叶变换计算得到。借助Mathematica软件中的GroebnerBasis命令,以 a, b, c, d 为变量,计算式(22)~(26)的Groebner基,得到关于 a 的一元四次方程如下:

$$k_4 a^4 + k_3 a^3 + k_2 a^2 + k_1 a + k_0 = 0 \quad (27)$$

式中, $k_i (i = 0, 1, 2, 3, 4)$ 为由目标轨迹谐波参数 $b'_n, \bar{b}'_n, c'_n, \bar{c}'_n$ 构成的已知量。求解方程(27)可得到 a 的4个非零解析解,将所得到的 a 解代入式(22)~(25)的方程组中,即可求得 b, c, d 的解,将所得结果代入方程(26)可求得变量 o 的解。在求得 a, b, c, d, o 解后,可由式(28)得到 $l_6, l_3, l_5, \alpha_1, \alpha_3$ 的解。

$$\begin{cases} l_6 = \pm \sqrt{ab}; \\ l_3 = \pm \sqrt{cd}; \\ l_5 = o; \\ \alpha_1 = -i \ln \frac{a}{l_8}; \\ \alpha_3 = -i \ln \frac{c}{l_3} - \beta \end{cases} \quad (28)$$

分析方程求解过程可知 $l_6, l_3, l_5, \alpha_1, \alpha_3$ 的解最终均可转化为仅含有 $c'_n, \bar{c}'_n, b'_n, \bar{b}'_n (n = -3, -2, \dots, 2, 3)$ 的计算公式,将其称为Stephenson-III型平面六杆机构右侧二杆组设计变量的计算通式。

由文献[14]可知, 左侧四杆机构的设计参数可以得到12组解, 将其与右侧二杆组的设计参数得到的4组解进行组合, 最终可以得到48组Stephenson-III型平面六杆机构轨迹综合设计参数。将综合所得机构代入仿真程序, 进行运动分析, 检验是否存在曲柄, 有无分支、顺序问题, 并依据综合误差, 最终可得到最优综合结果。

3 综合步骤

依据前面分析, 可以建立Stephenson-III型平面六杆机构轨迹综合的代数求解方法, 具体步骤如下:

1) 将Stephenson-III型平面六杆机构拆分为四杆机构和二级杆组, 根据E点的轨迹生成任务, 利用式(5)得到连杆曲线的谐波参数 c_n , 将其代入平面四杆机构轨迹综合的设计参数计算通用公式, 得到左侧

四杆机构的设计参数 $r, \mu, l_1, l_2, l_{21}, l_4, l_{61}, \varphi_0, \alpha_2, \beta_0$ 。

2) 由E点和P点轨迹坐标, 计算其对应点的转角 ρ_i , 根据 ρ_i 利用式(3)计算得到连杆转角函数 $e^{i\rho}$ 的谐波参数 b'_n , 利用式(13)~(15)计算得到 c'_n , 将 b'_n, c'_n 代入右侧二杆组设计变量的计算通式, 求解得到右侧二杆组设计参数 $l_6, l_3, l_5, \alpha_1, \alpha_3$ 。

3) 对所得48组综合结果进行运动仿真, 检验其是否存在曲柄, 有无分支问题、顺序问题, 并依据综合误差, 最终得到满足设计要求的Stephenson-III型平面六杆机构。

4 综合实例

实例: 综合Stephenson-III型平面六杆机构, 使其E点和P点实现的两条面包形轨迹, 目标轨迹的具体坐标值列于表1中。

表 1 E、P点轨迹采样点的坐标值

Tab. 1 Coordinates of prescribed points of E and P

序号	$\varphi_i(^{\circ})$	E_x	E_y	P_x	P_y	序号	$\varphi_i(^{\circ})$	E_x	E_y	P_x	P_y
1	0.00	39.03	131.26	47.98	230.85	33	180.00	-38.91	82.73	-28.56	182.19
2	5.63	46.27	141.02	59.13	240.18	34	185.63	-40.10	76.88	-31.17	176.48
3	11.25	52.76	149.44	69.29	248.06	35	191.25	-40.95	71.24	-33.44	170.96
4	16.88	58.13	156.52	77.96	254.53	36	196.88	-41.47	65.83	-35.40	165.65
5	22.50	62.14	162.36	84.85	259.75	37	202.50	-41.68	60.68	-37.06	160.57
6	28.13	64.71	167.10	89.83	263.90	38	208.13	-41.60	55.80	-38.42	155.75
7	33.75	65.86	170.90	92.94	267.16	39	213.75	-41.22	51.22	-39.50	151.20
8	39.38	65.68	173.86	94.30	269.68	40	219.38	-40.58	46.94	-40.32	146.94
9	45.00	64.32	176.08	94.08	271.55	41	225.00	-39.68	43.00	-40.89	143.00
10	50.63	61.91	177.61	92.46	272.82	42	230.63	-38.54	39.41	-41.21	139.38
11	56.25	58.60	178.48	89.65	273.53	43	236.25	-37.17	36.20	-41.30	136.11
12	61.88	54.55	178.72	85.82	273.70	44	241.88	-35.60	33.38	-41.16	133.22
13	67.50	49.89	178.34	81.16	273.32	45	247.50	-33.84	30.98	-40.79	130.73
14	73.13	44.76	177.35	75.83	272.40	46	253.13	-31.90	29.02	-40.22	128.67
15	78.75	39.26	175.77	69.96	270.94	47	258.75	-29.81	27.54	-39.43	127.08
16	84.38	33.50	173.61	63.69	268.95	48	264.38	-27.60	26.57	-38.44	125.98
17	90.00	27.60	170.89	57.14	266.43	49	270.00	-25.27	26.14	-37.24	125.42
18	95.63	21.64	167.63	50.41	263.40	50	275.63	-22.85	26.29	-35.83	125.45
19	101.25	15.69	163.86	43.61	259.88	51	281.25	-20.35	27.09	-34.19	126.12
20	106.88	9.84	159.61	36.81	255.90	52	286.88	-17.80	28.57	-32.32	127.51
21	112.50	4.16	154.92	30.10	251.50	53	292.50	-15.19	30.80	-30.18	129.67
22	118.13	-1.32	149.84	23.53	246.70	54	298.13	-12.53	33.87	-27.75	132.70
23	123.75	-6.52	144.41	17.18	241.56	55	303.75	-9.80	37.84	-24.94	136.69
24	129.38	-11.43	138.69	11.07	236.13	56	309.38	-6.97	42.83	-21.70	141.74
25	135.00	-15.99	132.73	5.26	230.44	57	315.00	-3.98	48.91	-17.90	147.94
26	140.63	-20.20	126.58	-0.24	224.57	58	320.63	-0.73	56.18	-13.41	155.37
27	146.25	-24.03	120.30	-5.38	218.55	59	326.25	2.91	64.69	-8.03	164.08
28	151.88	-27.47	113.95	-10.17	212.44	60	331.88	7.10	74.41	-1.59	174.03
29	157.50	-30.53	107.57	-14.58	206.29	61	337.50	12.01	85.20	6.10	185.03
30	163.13	-33.19	101.21	-18.63	200.15	62	343.13	17.76	96.78	15.12	196.74
31	168.75	-35.47	94.93	-22.30	194.06	63	348.75	24.34	108.68	25.36	208.67
32	174.38	-37.37	88.75	-25.61	188.06	64	354.38	31.56	120.35	36.50	220.22

1) 根据步骤2), 得到平面四杆机构连杆曲线的谐波参数 c_n , 如表2所示, 将上述所得的 c_n 的值代入四杆机构轨迹综合设计变量的计算通式, 可求得设计参数 r 、 μ 、 l_1 、 l_2 、 l_{21} 、 l_4 、 l_{61} 、 φ_0 、 α_2 、 β 。

2) 利用式(5)计算得到 c'_n , 计算转角 ρ_i , 进而得到连杆转角函数 $e^{i\rho}$ 的谐波参数 b'_n , 如表2所示, 将所得的 c'_n 、 b'_n 的值代入右侧二杆组设计变量的计算通式, 可求得设计参数 l_6 、 l_3 、 l_5 、 α_1 、 α_3 。

3) 将左侧四杆机构与右侧二杆组的设计参数进行组合得到48组解。应用仿真程序对所得综合结果进行运动分析和检验, 得到满足设计要求的一组机构参数为: $r = 5$, $l_1 = 75$, $l_{21} = 110$, $\mu = 0.349$, $\varphi_0 = 0.175$, $l_2 = 139.982$, $l_4 = 162.027$, $l_{61} = 160.008$, $\alpha_2 = 0.349$, $\beta = 0.174$, $l_6 = 263.886$, $l_3 = 201.871$, $l_5 = 161.421$, $\alpha_1 = 0.175$,

$\alpha_3 = 0.875$ 。

表 2 目标轨迹点的傅式级数展开的谐波参数

Tab. 2 Fourier coefficients of coordinates of prescribed points

谐波次数	c_n	c'_n	b'_n
-3	-1.680+3.245i	0.002+0.033i	0.002+0.010i
-2	-4.995+7.685i	-0.004+0.083i	0.004+0.032i
-1	-17.285+25.133i	-0.022+0.276i	0.004+0.108i
0	0.880+101.411i	0.423+0.802i	0.091+0.983i
1	57.265-4.441i	-0.241-0.177i	-0.015-0.113i
2	4.446-3.730i	0.018-0.050i	0.000-0.022i
3	0.734-0.424i	0.004-0.007i	0.001-0.005i

图2(a)、(b)分别为E、P两点目标轨迹与机构生成轨迹的比较。从图2中可以发现, 该方法综合的机构能够较好地再现目标轨迹。

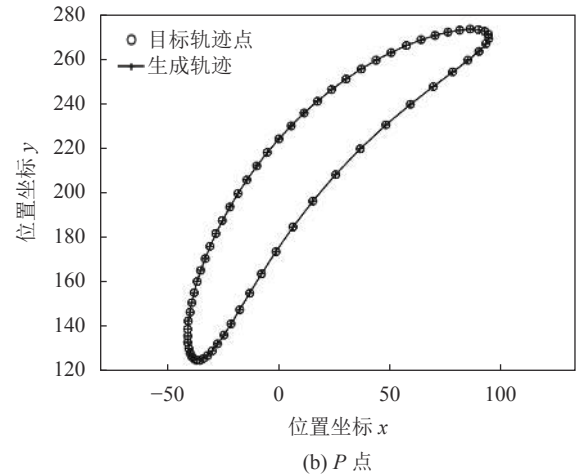
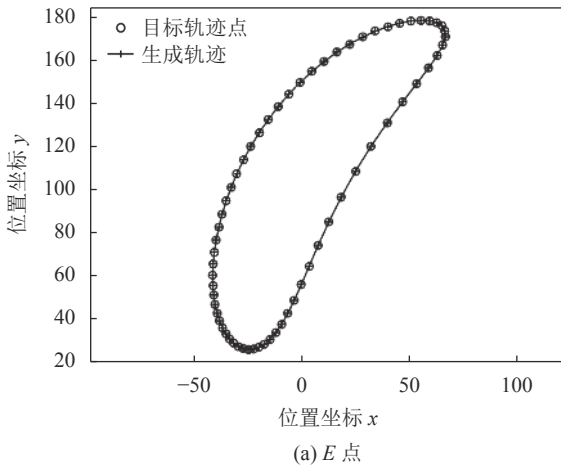


图 2 生成轨迹与目标轨迹点的比较

Fig. 2 Comparison between prescribed points and the corresponding generated path

5 结论

建立了一种基于傅氏级数的Stephenson-III型平面六杆机构轨迹综合的代数求解新方法。与已有的综合方法相比, 该方法克服了精确点数目的限制, 无需给定初值和建立数值图谱库, 不需要进行查找和迭代运算, 具有计算速度快、求解精度高、可重复性强的优点; 该方法可以同时提供多种综合结果, 为设计者提供更多的选择。如果该方法所得结果的精度不能满足设计要求, 可将其作为初值进行优化综合, 进一步得到满足设计要求的结果。

参考文献:

- [1] 梁崇高, 陈海宗. 平面连杆机构的计算设计[M]. 广州: 广东教育出版社, 1993.
- [2] Sun Jianwei, Wu Xin, Wang Jun, et al. Research on straight-lines mechanism path generation by using numerical atlas method[J]. *Mechanical Science and Technology for Aerospace Engineering*, 2005(6): 693-695. [孙建伟, 吴鑫, 王军, 等.

利用数值图谱法进行多杆直线导向机构的轨迹综合[J]. *机械科学与技术*, 2005(6): 693-695.]

- [3] 褚金奎. 连杆机构尺度综合的谐波特征参数法[M]. 北京: 科学出版社, 2010.
- [4] Bulatović R R, Đorđević S R. Optimal synthesis of a path generator six-bar linkage[J]. *Journal of Mechanical Science and Technology*, 2012, 26(12): 4027-4040.
- [5] Guo G, Zhang J, Gruver W A. Optimal design of a six-bar linkage with one degree of freedom for an anthropomorphic three-jointed finger mechanism[J]. *Proceedings of the Institution of Mechanical Engineers (Part H: Journal of Engineering in Medicine)*, 1993, 207(3): 185-190.
- [6] Tang Dunbing, Yang Jun, Dai Min. Object-oriented multiple poles and multi-objective constraint optimization design method[J]. *Machine Building & Automation*, 2016, 45(4): 1-4. [唐敦兵, 杨俊, 戴敏. 面向对象的多杆机构多目标约束优化设计方法[J]. *机械制造与自动化*, 2016, 45(4): 1-4.]

- [7] Li Hongzhong. Dimension synthesis of planar multi-bar linkages[J]. *China Science and Technology Information*, 2005(11):126. [李洪忠. 平面多杆机构的尺度优化综合[J]. *中国科技信息*, 2005(11):126.]
- [8] Wei Feng, Wei Shimin, Zhang Ying, et al. The algebraic solution for five precision points path synthesis of stephenson-III planar six-bar linkage[J]. *Journal of Beijing University of Posts and Telecommunications*, 2015, 38(5):104–108. [魏锋, 魏世民, 张英, 等. Stephenson-III 型平面六杆机构五精确点轨迹综合代数求解[J]. *北京邮电大学学报*, 2015, 38(5):104–108.]
- [9] Ma Jiayi. Singularity analysis of six degrees of freedom of parallel manipulators using geometric algebra[D]. Hangzhou: Zhejiang Sci-Tech University, 2016, 97:112–126. [马嘉熠. 基于几何代数的六自由度并联机构奇异分析[D]. 杭州: 浙江理工大学, 2017.]
- [10] Plecnik M M, McCarthy J M. Kinematic synthesis of Stephenson III six-bar function generators[J]. *Mechanism and Machine Theory*, 2016, 97:112–126.
- [11] Nafees K, Mohammad A. Dimensional synthesis of six-bar Stephenson III mechanism for 12 precision points path generation[J]. *International Journal of Mechanisms and Robotics*, 2016, 3(1):80–90.
- [12] Li Xuegang, Wei Shimin, Liao Qizheng, et al. The algebraic solution for path synthesis of planar five-bar linkages with timing[J]. *Journal of Beijing University of Posts and Telecommunications*, 2017, 40(1):23–27. [李学刚, 魏世民, 廖启征, 等. 平面五杆机构计时轨迹综合的代数求解[J]. *北京邮电大学学报*, 2017, 40(1):23–27.]
- [13] Li X G, Wei S M, Liao Q Z, et al. A novel analytical method for function generation synthesis of planar four-bar linkages[J]. *Mechanism and Machine Theory*, 2016, 101:222–235.
- [14] Li X G, Wei S M, Liao Q Z, et al. A novel analytical method for four-bar path generation synthesis based on Fourier series[J]. *Mechanism and Machine Theory*, 2020, 144:103671.
- [15] Li X G, Wu J, Ge Q J. A Fourier descriptor-based approach to design space decomposition for planar motion approximation[J]. *Journal of Mechanisms and Robotics*, 2016, 8(6):064501.
- [16] McGarva J, Mullineux G. Harmonic representation of closed curves[J]. *Applied Mathematical Modelling*, 1993, 17(4):213–218.

(编辑 黄小川)

引用格式: Li Xuegang, Zhang Lijuan, Wei Shimin, et al. Algebraic solution for path synthesis of planar Stephenson-III six-bar linkage[J]. *Advanced Engineering Sciences*, 2021, 53(1):155–161. [李学刚, 张丽娟, 魏世民, 等. Stephenson-III 型平面六杆机构轨迹综合的代数求解[J]. *工程科学与技术*, 2021, 53(1):155–161.]